

Five-Dimensional Gauged Supergravity Black Holes with Independent Rotation Parameters

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Abstract

We construct new non-extremal rotating black hole solutions in $SO(6)$ gauged five-dimensional supergravity. Our solutions are the first such examples in which the two rotation parameters are independently specifiable, rather than being set equal. The black holes carry charges for all three of the gauge fields in the $U(1)^3$ subgroup of $SO(6)$, albeit with only one independent charge parameter. We discuss the BPS limits, showing in particular that these include the first examples of regular supersymmetric black holes with independent angular momenta in gauged supergravity. We also find non-singular BPS solitons. Finally, we obtain another independent class of new rotating non-extremal black hole solutions with just one non-vanishing rotation parameter, and one non-vanishing charge.

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1 Introduction

Constructing non-extremal charged rotating black hole solutions in gauged supergravity is quite a complicated problem. This is because, unlike the case of ungauged supergravity, there are no known solution-generating techniques that could be used to add charges to the already-known neutral rotating black hole solutions found in four dimensions in [1], five dimensions in [2], and $D \geq 6$ dimensions in [3, 4]. Aside from the four-dimensional Kerr-Newman-AdS black holes, which were found in [5], the known non-extremal charged rotating black hole solutions comprise recently-discovered examples in five-dimensional gauged supergravities in [6, 7]; in four-dimensional gauged supergravity in [8]; and in seven-dimensional gauged supergravity in [9]. In the five and seven-dimensional cases, the problem was simplified greatly by taking the *a priori* independent rotation parameters of the orthogonal 2-planes in the transverse space to be equal. This reduces the problem to studying cohomogeneity-1 metrics, with non-trivial coordinate dependence on only the radial variable, rather than metrics of cohomogeneity 2 or cohomogeneity 3.

In this paper, we shall present some new results on non-extremal rotating black holes in five-dimensional gauged supergravity, in which the two rotation parameters a and b can be independently specified. Our black holes can be viewed as solutions in $N = 8$ gauged $SO(6)$ supergravity, with three charge parameters associated with the gauge fields of the $U(1)^3$ abelian subgroup. They can also be viewed as solutions in $N = 2$ gauged supergravity coupled to two vector multiplets. Our solutions are not the most general possible, in that there is a specific relation between the three charges. However, the metrics have considerable interest because they allow us to study the thermodynamics, and the supersymmetric limits, for a rather general class of non-extremal black holes with unequal angular momenta.

After presenting the solutions, we then calculate the charges, angular momenta, angular velocities, electrostatic potentials, temperature and entropy. From these, we follow the procedure that was used in [10], and more recently in [11], for calculating the energy by integration of the first law of thermodynamics. The fact that this is possible at all, i.e. that the right-hand side of the first law is an exact differential, is already a highly non-trivial test of the validity of the thermodynamic relations for these black holes.

Having obtained expressions for the energy E , the angular momenta J_a and J_b , and the three charges Q_i , we can study the conditions under which supersymmetric limits will arise, by looking for zero eigenvalues of the Bogomol'nyi matrix arising from anticommutators of the supercharges. We obtain by this means families of supersymmetric configurations, characterised by a mass parameter and the two independent rotation parameters. In general

these BPS solutions have naked closed timelike curves (CTC's) lying outside a Killing horizon. However, for a particular choice of the mass, we obtain completely regular black holes with no singularities or closed timelike curves on or outside the horizon. These are similar to the regular black holes of five-dimensional gauged supergravity that were found in [12, 13], except that in our new solutions the two angular momenta can be independently specified. Indeed, the rotating BPS black holes that we find in this paper are the first such examples with independent rotation parameters. We also find other special cases, describing completely regular solitons.

We also obtain a further class of new non-extremal rotating black hole solutions of five-dimensional gauged supergravity, in which only one rotation parameter is non-vanishing, and only one of the three $U(1)$ charges is turned on. These solutions are therefore independent of any found previously in this paper or elsewhere. We again study the thermodynamics and obtain expressions for the conserved energy, angular momentum and charge. From the BPS limit we again obtain supersymmetric solutions. In this case, unlike the one discussed above, there are no regular BPS black holes or solitons, but only solutions with naked CTC's.

2 Black Holes with Two Unequal Rotations

The rotating black hole metrics that we shall construct arise as solutions of $SO(6)$ gauged five-dimensional supergravity. They are charged under the $U(1)^3$ Cartan subgroup of $SO(6)$, with specific relations between the three charges. The two rotation parameters can be specified independently. The relevant part of the supergravity Lagrangian that describes these solutions is given by

$$e^{-1} \mathcal{L} = R - \frac{1}{2} \partial \vec{\varphi}^2 - \frac{1}{4} \sum_{i=1}^3 X_i^{-2} (F^i)^2 + 4g^2 \sum_{i=1}^3 X_i^{-1} + \frac{1}{24} \epsilon_{ijk} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^i F_{\rho\sigma}^j A_{\lambda}^k, \quad (1)$$

where $\vec{\varphi} = (\varphi_1, \varphi_2)$, and

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \quad (2)$$

In the following subsections we shall first construct the non-extremal rotating metrics, and then calculate their associated conserved quantities, namely their mass E , their angular momenta J_a and J_b , and the three electric charges Q_i . Next, by using the BPS conditions derived from the AdS superalgebra, we determine the restrictions on the parameters of the solutions that lead to supersymmetry. We investigate the global structure of these BPS

limits, showing in particular that there exist regular supersymmetric black holes with no naked singularities or closed timelike curves.

2.1 The Non-Extremal Black Holes

Since there are no solution-generating techniques available for constructing non-extremal rotating black holes in gauged supergravities, our procedure for obtaining them depends to a large extent on a combination of guesswork and conjecture, followed by an explicit verification that the equations of motion are indeed satisfied. Here, we simply present the outcome of this process.

We find that the following provides a solution of the five-dimensional gauged supergravity equations:

$$\begin{aligned}
ds^2 &= H^{-\frac{4}{3}} \left[-\frac{X}{\rho^2} (dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - b \cos^2 \theta \frac{d\psi}{\Xi_b})^2 \right. \\
&\quad + \frac{C}{\rho^2} \left(\frac{ab}{f_3} dt - \frac{b}{f_2} \sin^2 \theta \frac{d\phi}{\Xi_a} - \frac{a}{f_1} \cos^2 \theta \frac{d\psi}{\Xi_b} \right)^2 \\
&\quad \left. + \frac{Z \sin^2 \theta}{\rho^2} \left(\frac{a}{f_3} dt - \frac{1}{f_2} \frac{d\phi}{\Xi_a} \right)^2 + \frac{W \cos^2 \theta}{\rho^2} \left(\frac{b}{f_3} dt - \frac{1}{f_1} \frac{d\psi}{\Xi_b} \right)^2 \right] + H^{\frac{2}{3}} \left(\frac{\rho^2}{X} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \right), \\
H &= \tilde{\rho}^2 / \rho^2, \quad \rho^2 = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \quad \tilde{\rho}^2 = \rho^2 + 2m s^2, \\
f_1 &= a^2 + r^2, \quad f_2 = b^2 + r^2, \quad f_3 = (a^2 + r^2)(b^2 + r^2) + 2m r^2 s^2; \\
\Delta_\theta &= 1 - a^2 g^2 \cos^2 \theta - b^2 g^2 \sin^2 \theta, \\
X &= \frac{1}{r^2} (a^2 + r^2)(b^2 + r^2) - 2m + g^2 (a^2 + r^2 + 2m s^2)(b^2 + r^2 + 2m s^2), \\
C &= f_1 f_2 (X + 2m - 4m^2 s^4 / \rho^2), \\
Z &= -b^2 C + \frac{f_2 f_3}{r^2} [f_3 - g^2 r^2 (a^2 - b^2)(a^2 + r^2 + 2m s^2) \cos^2 \theta], \\
W &= -a^2 C + \frac{f_1 f_3}{r^2} [f_3 + g^2 r^2 (a^2 - b^2)(b^2 + r^2 + 2m s^2) \sin^2 \theta], \\
\Xi_a &= 1 - a^2 g^2, \quad \Xi_b = 1 - b^2 g^2,
\end{aligned} \tag{3}$$

where

$$s \equiv \sinh \delta, \quad c \equiv \cosh \delta. \tag{4}$$

The gauge potentials and scalar fields are given by

$$\begin{aligned}
A^1 &= A^2 = \frac{2m s c}{\tilde{\rho}^2} (dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - b \cos^2 \theta \frac{d\psi}{\Xi_b}) \\
A^3 &= \frac{2m s^2}{\rho^2} (b \sin^2 \theta \frac{d\phi}{\Xi_a} + a \cos^2 \theta \frac{d\psi}{\Xi_b}) \\
X_1 &= X_2 = H^{-\frac{1}{3}}, \quad X_3 = H^{\frac{2}{3}}.
\end{aligned} \tag{5}$$

It should be noted that the solution above is presented in a coordinate frame that is rotating at infinity. One can pass to coordinates that are asymptotically static by making the redefinitions

$$\phi = \tilde{\phi} + ag^2 t, \quad \psi = \tilde{\psi} + bg^2 t. \quad (6)$$

It is helpful to make this transformation in order to simplify the calculation of the thermodynamic quantities. One might think from the expressions for the gauge potentials in (5) that there are just two non-vanishing (and equal) charges, since A^3 has no electric component. However, this is a somewhat misleading artefact of the original rotating coordinate system. After transforming to the asymptotically non-rotating frame, one finds that A^3 also has an electric component, and indeed, as we shall see below, the third electric charge is non-zero too.

It is straightforward to calculate the temperature, entropy, angular velocities on the horizon, and the electrostatic potentials on the horizon, referred to the asymptotically static frame. We find

$$\begin{aligned} T &= \frac{2g^2 r_+^6 + [1 + g^2(a^2 + b^2 + 4ms^2)]r_+^4 - a^2b^2}{2\pi r_+[r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2]}, \\ S &= \frac{\pi^2[r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2]}{2\Xi_a \Xi_b r_+}, \\ \Omega_a &= \frac{a[g^2 r_+^4 + (b^2 + 2ms^2)g^2 r_+^2 + b^2]}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\ \Omega_b &= \frac{b[g^2 r_+^4 + (a^2 + 2ms^2)g^2 r_+^2 + a^2]}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\ \Phi_1 &= \Phi_2 = \frac{2mr_+^2 sc}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \\ \Phi_3 &= \frac{2mabs^2}{r_+^4 + (a^2 + b^2 + 2ms^2)r_+^2 + a^2b^2}, \end{aligned} \quad (7)$$

where r_+ , the largest root of the metric function $X(r)$, is the location of the outer horizon.

The two angular momenta can be evaluated from Komar integrals

$$J = \frac{1}{16\pi} \int_{S^3} *dK, \quad (8)$$

where K is the 1-form obtained by lowering the index on the angular Killing vector $\partial/\partial\phi$ or $\partial/\partial\psi$. The charges are given by Gaussian integrals

$$Q_i = \frac{1}{16\pi} \int_{S^3} (X_i^{-2} *F^i - \frac{1}{2}\epsilon_{ijk} A^j \wedge A^k), \quad (9)$$

Having evaluated the angular momenta and charges, we can then integrate the first law of

thermodynamics

$$dE = TdS + \Omega_a dJ_a + \Omega_b dJ_b + \sum_i \Phi_i dQ_i \quad (10)$$

in order to obtain the energy E of the black hole solution. Our results for the conserved quantities are

$$E = \frac{\pi m [2\Xi_a + 2\Xi_b - \Xi_a \Xi_b + (2\Xi_a^2 + 2\Xi_b^2 + 2\Xi_a \Xi_b - \Xi_a^2 \Xi_b - \Xi_b^2 \Xi_a) s^2]}{4\Xi_a^2 \Xi_b^2}, \quad (11)$$

$$J_a = \frac{\pi m a (1 + s^2 \Xi_b)}{2\Xi_b \Xi_a^2}, \quad J_b = \frac{\pi m b (1 + s^2 \Xi_a)}{2\Xi_a \Xi_b^2},$$

$$Q_1 = Q_2 = \frac{\pi m s c}{2\Xi_a \Xi_b}, \quad Q_3 = -\frac{\pi a b m s^2 g^2}{2\Xi_a \Xi_b}. \quad (12)$$

2.2 The BPS limit

As discussed recently in [11], the BPS limit of the non-extremal solution can conveniently be discussed by studying the eigenvalues of the Bogomol'nyi matrix that arises from the anticommutator of the supercharges of the AdS superalgebra. Thus the BPS limit of non-extremal five-dimensional gauged supergravity solutions is attained when¹

$$E + gJ_a + gJ_b - \sum_i Q_i = 0. \quad (13)$$

Substituting our expressions for the conserved mass, angular momenta and charges given in (12), we find that the BPS condition is satisfied if the parameter δ is chosen so that

$$e^{2\delta} = 1 + \frac{2}{(a+b)g}. \quad (14)$$

In section (2.3), we shall discuss the global structure of the BPS solutions.

It is interesting to note that with a and b as independently specificable parameters, we can make contact with previous results in two inequivalent special cases. Firstly, if we take $a = b$, the solutions we have obtained in this paper reduce to particular cases of the 3-charge rotating black holes with equal angular momenta that were found in [7]. In particular, the BPS condition (14) reduces to one that was found for the $a = b$ solutions in [11]. An inequivalent special case arises if instead we take $a = -b$. Now, the BPS condition (14) reduces to the condition that $e^{2\delta} \rightarrow \infty$, which was also arose, as a disjoint case, in the analysis in [11]; it again can be viewed as a situation with “equal angular momenta,” after making an orientation reversal. Because in the present work we have the possibility to

¹The Bogomol'nyi matrix actually has four eigenvalues, namely $E \pm gJ_a \pm gJ_b - \sum_i Q_i$. They are equivalent, under reversal of the signs of the rotation parameters, and so without loss of generality we have chosen just to consider one of them.

specify a and b independently, we can actually describe a continuous interpolation between two BPS limits that were seen as disjoint possibilities in the earlier work.

2.3 Global analysis

To analyse the global structure of the metric (3), we first rewrite it in the form

$$ds^2 = H^{2/3} \left[-\frac{X \Delta_\theta \sin^2 2\theta}{4\Xi_a \Xi_b H^2 B_\phi B_\psi} r^2 dt^2 + \rho^2 \left(\frac{dr^2}{X} + \frac{d\theta^2}{\Delta_\theta} \right) + B_\psi (d\psi + v_1 d\phi + v_2 dt)^2 + B_\phi (d\phi + v_3 dt)^2 \right]. \quad (15)$$

From this, it is evident that there is an outer Killing horizon located at $r = r_+$, the largest root of $X(r)$. There is a velocity of light surface (VLS), located at the boundary $r = r_L$ of the region where $B_\psi B_\phi$ changes sign from positive (at large r) to negative. Inside the VLS, the metric develops closed timelike curves (CTC's). If $r_+ > r_L$, then the Killing horizon lies outside the VLS, and so the Killing horizon is an event horizon. In these circumstances, the solution describes a regular black hole, in which there are neither curvature singularities nor CTC's outside the horizon. If the largest root r_+ is inside the VLS, the solution instead describes a naked time machine.

In the supersymmetric limit, there exists a Killing vector

$$\ell = \frac{\partial}{\partial t} - g \frac{\partial}{\partial \tilde{\phi}} - g \frac{\partial}{\partial \tilde{\psi}} \quad (16)$$

that has a spinorial square root, in the sense that $\ell \sim \bar{\eta} \gamma^\mu \eta \partial_\mu$, where η is the Killing spinor. (See [11] for a recent discussion of this.) This Killing vector is necessarily non-spacelike, and in fact we find that the explicit expression for its norm is a manifestly non-positive quantity. From this, we find the identity

$$\begin{aligned} & -\frac{X \Delta_\theta \sin^2 2\theta}{4\Xi_a \Xi_b H^2 B_\psi B_\phi} + B_\psi (v_2 + g - bg^2 + v_1(g - ag^2))^2 + B_\phi (v_3 + g - ag^2)^2 \\ & = -\frac{\left[(1 + ag + bg)(1 + ag \cos^2 \theta + bg \sin^2 \theta) - H(ag(1 + ag) \cos^2 \theta + bg(1 + bg) \sin^2 \theta) \right]^2}{(1 + ag)^2 (1 + bg)^2 H^2} \end{aligned} \quad (17)$$

It follows from (17) that in general we shall have $B_\phi B_\psi < 0$ at the largest root $r = r_+$ where $X(r)$ vanishes. This means that the VLS at $r = r_L$ lies outside the Killing horizon at $r = r_+$, thus implying naked CTC's.

As in the cases discussed in [11], there are two ways of avoiding such naked CTC's. The first is if the parameters are chosen so that right-hand side of (17) vanishes at $r = r_+ \equiv r_0$. This occurs if

$$m = \frac{(a+b)^2(1+ag)(1+bg)(2+ag+bg)}{2(1+ag+bg)}. \quad (18)$$

Note that when this condition is satisfied, the function X becomes

$$X = \frac{(r - r_0)^2((ab - r_0)^2 r^2 + a^2 b^2 (a + b)^2)}{(a + b)^2 r_0^4 r^2}, \quad (19)$$

and there is in fact a double root at $r = r_0$, where r_0 is given by

$$r_0^2 = \frac{ab}{1 + ag + bg}. \quad (20)$$

Now it is straightforward to verify that the VLS lies inside the horizon at $r = r_0$, and so the Killing horizon is an event horizon. Thus the solution describes a supersymmetric black hole that is regular on and outside the event-horizon. The fact that $X(r)$ has a double root at $r = r_0$ implies that the Hawking temperature is zero. This is the first example of a supersymmetric black hole with two independent angular momenta in gauged supergravity. If a and b are set equal, the solution reduces to a special case of the regular black holes found in [13].² The rotation parameters a and b must in general be restricted to an appropriate range, in order to ensure that B_ϕ and B_ψ remain positive and hence that there are no CTC's outside the horizon.

The other way to avoid the naked CTC's of the generic supersymmetric solutions is by restricting the parameters so that $B_\psi B_\phi$ goes to zero at the same radius as X goes to zero, i.e. so that $r_L = r_+$. This occurs when the parameter m is given by

$$m = \frac{(a + b)(1 + ag)(1 + bg)(2 + ag + bg)(1 + 2ag + bg)(1 + ag + 2bg)}{2g(1 + ag + bg)^2}. \quad (21)$$

The solution then describes a smooth topological soliton. Defining $R = r^2 + a^2 b^2 g^2 / (1 + ag + bg)^2$, the coordinate R runs from 0 to ∞ . The requirement of having no conical singularity at $R = 0$, where $B_\phi = 0$, implies the quantisation condition

$$\frac{1 + 3(a + b)g + (3a^2 + 5ab + 3b^2)g^2 + (a + b)(a^2 + b^2)g^3 - ab(a^2 + 4ab + b^2)^4}{bg(1 - ag)(1 + ag + bg)(1 + 2ag + bg)} = 1. \quad (22)$$

In the special cases $a = b$ or $a = -b$, these topological solitons are encompassed within the soliton solutions obtained in [11]. The rotation parameters a and b must in general be restricted to an appropriate range, in order to ensure that B_ϕ and B_ψ remain positive and hence that there are no CTC's for all $R \geq 0$.

Aside from the above two possibilities, the supersymmetric solutions have naked CTC's in general. As in the examples in [11], a conical singularity at the Killing horizon can

²Note that a black hole is not possible if $a + b = 0$. As we discussed in section 2.2, setting $a = -b$ corresponds to a BPS limit that was also studied in [11], which was associated with BPS solutions found in [14, 15]. The solutions in [14, 15] all describe configurations with naked CTC's, or, as shown in [11], a non-singular soliton; there is no solution in that family that describes a regular black hole.

be avoided by periodically identify the asymptotic time coordinate t with an appropriate period. However, if the Killing horizon is associated with a double root of $X(r)$, then such an identification is unnecessary.

3 Black Holes with One Rotation and One Charge

In this section we obtain another new solution describing a non-extremal rotating black hole in gauged five-dimensional supergravity. In this case, just one of the two rotation parameters is non-zero, and only one of the three gauge fields in the $U(1)^3$ subgroup of $SO(6)$ is turned on. This solution is therefore not a special case of the solution obtained in section 2, nor indeed of any other previously-obtained solutions. Having presented the solution, we then evaluate the conserved mass, angular momentum and charge, and from this we study the BPS limit.

3.1 The non-extremal solution

Again, since the process that led us to the solution was a long one, involving making a conjecture for its form, and then verifying explicitly that it solved the equations, we shall just present our final result here. We find that the metric for the five-dimensional black hole with one non-vanishing rotation parameter and one charge is given by

$$\begin{aligned}
ds^2 &= -H^{-\frac{2}{3}} \frac{wY}{F(r, \theta)} (c dt - a \sin^2 \theta \frac{d\phi}{w\Xi})^2 \\
&+ H^{\frac{1}{3}} \left(\frac{w\Delta_\theta \sin^2 \theta}{F(r, \theta)} (f_1 dt - c f_2 \frac{d\phi}{w\Xi})^2 + \frac{\rho^2}{Y} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + r^2 \cos^2 \theta d\psi^2 \right), \\
F(r, \theta) &= c^2 f_2(r) - a \sin^2 \theta f_1(r), \\
H &= \tilde{\rho}^2 / \rho^2, \quad \tilde{\rho}^2 = a^2 + r^2 + 2ms^2, \quad \rho^2 = a^2 + r^2, \\
f_1 &= \frac{a}{w} (1 - w s^2 g^2 (a^2 + r^2)), \quad f_2 = w (a^2 + r^2), \\
Y &= (a^2 + r^2)(1 + g^2 r^2) - 2m f_1 / a, \\
\Delta_\theta &= 1 - a^2 g^2 \cos^2 \theta, \quad \Xi = 1 - a^2 g^2.
\end{aligned} \tag{23}$$

where again $c = \cosh \delta$, $s = \sinh \delta$, and the constant w , which satisfies $c^2 w^2 - s^2 w \Xi = 1$, is given by

$$w = \frac{\Xi s^2 + \sqrt{4(1 + s^2) + \Xi^2 s^4}}{2(1 + s^2)}. \tag{24}$$

The gauge potentials and scalar fields are given by

$$A^1 = \frac{2ms\sqrt{w}}{\tilde{\rho}^2} (c dt - a \sin^2 \theta \frac{d\phi}{w\Xi}), \quad A^2 = A^3 = 0,$$

$$X_1 = H^{-\frac{2}{3}}, \quad X_2 = X_3 = H^{\frac{1}{3}}. \quad (25)$$

Following an analysis analogous to the one we employed in section 2.1, we find that the conserved energy, angular momentum and charge for this black hole solution are given by

$$\begin{aligned} E &= \frac{\pi m [\Xi - w(2 + \Xi) + w^2 \Xi (1 + \Xi)]}{4 \Xi^2 w (\Xi - w)}, \\ J &= \frac{\pi m a \sqrt{1 - w \Xi}}{2 \Xi^2 \sqrt{w(w - \Xi)}}, \\ Q &= \frac{\pi m \sqrt{(1 - w^2)(1 - w \Xi)}}{2 \Xi \sqrt{w} (\Xi - w)}. \end{aligned} \quad (26)$$

3.2 The BPS limit

The supersymmetry condition following from the requirement of a vanishing eigenvalue for the Bogomol'nyi matrix is now given (modulo equivalent sign choices) by

$$E - g J - Q = 0. \quad (27)$$

Using our expressions (26) for the conserved energy, angular momentum and charge, we find that this is BPS condition implies

$$a^2 g^2 = \frac{1 - w}{w^2}. \quad (28)$$

Alternatively, it can be expressed as

$$ag = \frac{c}{s^2}. \quad (29)$$

To analyse the global properties of the solution, it is helpful to rewrite the metric as

$$ds^2 = H^{1/3} \left(-\frac{Y \Delta_\theta \sin^2 \theta dt^2}{(1 - a^2 g^2)^2 H B_\phi} + \frac{\rho^2 dr^2}{Y} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + B_\phi (d\phi + v dt)^2 + r^2 \cos^2 \theta d\psi^2 \right). \quad (30)$$

This expression is valid both in the BPS limit and in the non-extremal case. In the supersymmetric limit there exists a Killing vector $\ell = \partial/\partial t - g\partial/\partial\tilde{\phi} - g\partial/\partial\psi$ with a spinorial square root, where $\tilde{\phi} = \phi + ag^2 w c t$, and $(\tilde{\phi}, \psi)$ are asymptotically non-rotating coordinates. From the expression for the norm of ℓ , which is manifestly non-positive, we can read off the identity

$$-\frac{Y \Delta_\theta \sin^2 \theta}{(1 - a^2 g^2)^2 H B_\phi} + B_\phi (v - ag^2 w c + g)^2 + r^2 \cos^2 \theta g^2 = -H^{-1}. \quad (31)$$

Thus in general at the Killing horizon, where $X = 0$, either B_ϕ or r^2 is negative, implying the existence of naked CTC's. It is straightforward to verify, unlike the previous example in section 2, here naked CTC's are unavoidable when there is only one charge and one non-vanishing rotation. The BPS solutions therefore all describe naked time machines.

4 Conclusion

In this paper, we have constructed new non-extremal black hole solutions in five-dimensional $SO(6)$ gauged supergravity. The new solutions go beyond what has been found previously, by having unequal values for the angular momenta in the two orthogonal 2-planes in the transverse space. This means that the metrics are considerably more complicated, since they are of cohomogeneity 2, rather than the cohomogeneity 1 of all the previously known examples.

We have found two classes of solutions. In the first, the two rotation parameters are independently specifiable, as also are the mass, and a parameter that characterises the three electric charges carried by the gauge fields of the abelian $U(1)^3$ subgroup of the $SO(6)$ gauge group. Having obtained the non-extremal black hole solutions, we then calculated the conserved angular momenta and charges, and, by integrating the first law of thermodynamics, the energy. Using these results we then studied the BPS limits that give rise to supersymmetric backgrounds. In general, the BPS solutions have closed timelike curves outside a Killing horizon, and hence they describe “naked time machines.” However, for a special choice of the relation between the mass and the rotation parameters, we obtain a completely regular black hole, with neither CTC’s nor singularities outside the event horizon. Since the two rotation parameters still remain as free parameters, these black hole solutions provide a continuous supersymmetric interpolation between certain previously obtained equal-rotation solutions. Our new solutions provide the first examples of supersymmetric black holes in gauged supergravity in which there are independent rotation parameters. We also find, in another special case, a solution describing a completely non-singular soliton.

We have also found a second new class of non-extremal black hole solutions, independent of the first class, in which just one rotation parameter is non-zero, and only one of the three $U(1)$ charges is non-vanishing. We again calculated the conserved angular momentum, charge and energy, and studied the BPS limits. In this case, we find that there are no regular supersymmetric black holes or solitons, but rather, the BPS solutions describe backgrounds with closed timelike curves outside a Killing horizon.

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